2 Analysis of electromagnetism in a single-phase transformer

The goal of the second assignment is to continue the study of the single phase transformer and to focus on the electromagnetic energy conversion from the primary (excitation or generating) coil to the secondary (load or motoring) coil. There are two models based on equivalent circuit method (ECM) with the focus on the magnetic core (magnetic circuit) and the electric equivalent circuit of the transformer and two models based on finite element method (FEM) with DC and AC currents. In this assignment use equivalent circuit analysis (ECA) and finite element analysis (FEA) in order to bring your attention towards

- Implementation of nonlinear core model in ECM
- Transformer equations and equivalent circuit representation
- Definition and extraction of transformer parameters by analytical expressions and FEA
- Transformer characteristics by electric ECA driven by AC potential difference
- Magnetostatic and quasi static electromagnetic FEA driven by AC-currents.
- Power capability estimation by heat transfer analysis vs electromagnetic energy conversion.

2.1 Getting started

The goal of this assignment is to study the power conditioning requirements and the transformer characteristics by solving the set of equations describing the electric and magnetic circuits. In essence the common purpose of the transformer is

- Power transfer
- Power conditioning
- Galvanic separation

Apart from the transfer power capability, the importance of the transformer is to provide an output voltage that is adapted to the load, which is usually different from the supply voltage. The essential part of the power conditioning is to become aware of the voltage drop(s) across the transformer, which is similar to the heat power drop(s) across the transformer in the thermal analysis. Therefore the output of the transformer is limited either magnetically or thermally. Magnetic limitation is associated to magnetic saturation, flux linkage and flux leakage. The transformer model is defined by equivalent circuit method and finite element method. The way of solving electrical equivalent circuit (EEC) or magnetic equivalent circuit (MEC) is quite a same as solving the thermal equivalent circuit (TEC). Therefore the program structure is quite similar to the one handled previously including the same transformer type and the geometric conditions, the same parameterisation and the same model setup. Anyhow the crucial part is the parameter estimation, where the challenges with the FEA are introduced. It is recommended to use the same dimensions as in the previous assignment and use proportion parameter $k_s=0.5$. This parameter, $k_s$, defines the geometric proportion between the slot length and core limb length.

2.2 Equivalent circuit of a transformer

Equivalent circuit facilitates to analyze the electromagnetic behaviour of the transformer, to consider the voltage drops in it and to study the power conditioning requirements: nominal output voltage and voltage regulation. The electric equivalent circuit model considers the magnetically coupled primary and secondary circuits driven by input voltage. There is also magnetic equivalent
circuits, which is defined according to Ampere’s law for the closed contour and this is defined as current driven equation similar to the FEA:

\[
HI = \frac{B}{\mu} l = \phi \frac{l}{\mu A} = \phi R = \phi \frac{1}{P} = NI
\]  \tag{2.1}

In this equation the parameters \( l \) and \( A \) defines the magnetising path and cross-sectional area of the magnetic core. The primary concern is related to the magnetisation characteristics and magnetic saturation.

### 2.3 Ideal transformer

Ideal transformer describes the ideal electromagnetic coupling with no losses or voltage/MMF drop in the electric/magnetic circuits. There are no other magnetic fluxes than just magnetising flux

\[\phi = \phi_{12} = \phi_{21}\]  \tag{2.2}

MMF is abbreviation of magnetomotive force that is the driving force of the magnetic flux in the magnetic circuit like electromotive force (EMF) is the driving force of the electric current in electric circuits. The electrical equations in the ideal transformer are

\[
\begin{aligned}
\frac{du_1}{dt} &= N_1 \frac{d\phi}{dt} \\
\frac{du_2}{dt} &= N_2 \frac{d\phi}{dt}
\end{aligned}
\]  \tag{2.3}

The magnetic equations

\[
\begin{cases}
\phi = PN_1i_1 \\
\phi = PN_2i_2
\end{cases}
\]  \tag{2.4}

where for the ideally coupled windings the primary MMF balances the secondary MMF. The mutual flux \( \phi \) is the means of transfer of energy from primary to secondary, and links both windings. In an ideal transformer, this flux requires negligibly small ampere-turns to produce it, so the net ampere-turns, primary plus secondary, are about zero. When a current is drawn from the secondary in the positive direction, ampere-turns decrease substantially. This must be matched by an equal increase in primary ampere-turns, which is caused by an increase in the current entering the primary in the positive direction. In this way, the back-EMF of the primary (the voltage induced in it by the flux \( \phi \)) equals the voltage applied to the primary, as it must. The power conditioning is achieved by defining the number of winding turns for the primary and the secondary winding, where the transformation coefficient \( n \) is in accordance with instantaneous power balance

\[n = \frac{u_2}{u_1} = \frac{N_2}{N_1} = \frac{i_1}{i_2}\]  \tag{2.5}

If an impedance \( Z_2 \) is connected to the secondary then \( U_2/I_2 = Z_2 \) and considering the relations between the primary and the secondary voltages and currents then the impedance \( Z_1 \) by the primary terminals is

\[Z_1 = \frac{U_1}{I_1} = \frac{1}{n^2} \frac{U_2}{I_2} = \frac{1}{n^2} Z_2\]  \tag{2.6}

that is the transformer property of changing impedances and this advantage is used for impedance matching.

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2.4 Real transformer

In the real transformer, the winding resistances are not zero, the magnetic coupling between the coils is not perfect and the reluctance of the magnetic core is not zero. Due to the imperfect magnetic coupling there is magnetic leakage apart from the magnetic linkage.

\[ \phi_1 - \phi_{\sigma 1} = \phi_1 - (1 - k)\phi_1 = k\phi_1 = \phi_{21} = \phi_{12} = k\phi_2 = \phi_2 - (1 - k)\phi_2 = \phi_2 - \phi_{\sigma 2} \quad (2.7) \]

The magnetically coupled flux \( \phi = \phi_2 = \phi_{\sigma 1} \) is the same for the magnetically coupled coils. These fluxes linked with the number of turns define flux linkages and inductances – self inductance, coupled flux current relation – mutual inductance and the uncoupled leakage inductance. For the sake of convenience the coupling coefficient \( k \) defines the ratio between linking and leaking magnetic fluxes and the corresponding inductances of a coil self inductance

\[ L_m = kL_1 \quad (2.8) \]
\[ L_{\sigma 1} = (1 - k)L_1 \quad (2.9) \]

Including the real flux paths of the coils, the formulation of the ideal transformer becomes

\[
\begin{align*}
\frac{di_1}{dt} &= R_i i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\
\frac{di_2}{dt} &= -R_2 i_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}
\end{align*} \quad (2.10) 
\]

Where magnetic equations are

\[
\begin{align*}
\phi_1 &= \phi_{\sigma 1} + \phi_{21} = \frac{L_{\sigma 1} i_1}{N_1} + \frac{M i_1}{N_2} = \frac{L_1 i_1}{N_1} \\
\phi_2 &= \phi_{\sigma 2} + \phi_{12} = \frac{L_{\sigma 2} i_2}{N_2} + \frac{M i_2}{N_1} = \frac{L_2 i_2}{N_2}
\end{align*} \quad (2.11) 
\]

This treats the windings as a pair of mutually coupled coils with both primary and secondary windings passing currents. The negative sign in these equations arise from the reversed direction of the secondary current \( i_2 \). From the transformer equation, the primary MMF must equal the secondary MMF, and since these are in opposite directions, in an ideal transformer they cancel so that there is no overall resultant flux in the core. This can be seen as any unopposed primary EMF would create a large primary current and therefore a large flux in the core due to the primary winding. However, this large flux would necessarily cause a large current to flow in the secondary circuit and this current must create an opposing flux that effectively cancels the initiating primary flux. In a non-ideal transformer, the resultant flux in the core is needed to magnetise the core. This is called the magnetising flux. Apart from the coupling flux the total flux linkage of the coil includes also a leakage flux that does not couple with the secondary coil. By describing the flux over current elements as inductances the imperfection of the real magnetic coupling can be taken into the consideration.

\[
\begin{align*}
L_1 &= L_{\sigma 1} + \frac{N_1}{N_2} M \\
L_2 &= L_{\sigma 2} + \frac{N_2}{N_1} M
\end{align*} \quad (2.12) 
\]

after the substitutions of the ‘description’ of magnetic circuit into the electrical equations the system equation becomes

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The further development of the circuit to become equivalent to the real transformer gives Figure 2.2.

Figure 2.1 Circuit representation for the equation system of a transformer

Figure 2.2 Complete equivalent circuit for a transformer includes the effects of shunt and inter-winding capacitance, stray inductance, magnetic loss, and winding resistance. Under low frequency or large-signal conditions, the shunt primary inductance can become nonlinear if the transformer is driven into saturation.

Figure 2.2 indicates the complete equivalent circuit that is equally applicable to a pulse and a wideband transformer. This results that the circuit can be used in the analysis that focuses on time domain (pulse transformers) rather than on frequency domain (band transformers). The particular difference between Figure 2.1 and Figure 2.2 is that the first circuit neglects the capacitance effects between the turns and between the windings, magnetic non-linearity is ignored and iron losses are taken into account by introducing a parallel resistance $R_c$ besides the magnetising inductance $L_m$. For a given transformer, the values of the equivalent circuit elements are same for the pulse and the wideband application.

2.5 **Specification of circuit elements**

The circuit elements determine the electromagnetic behaviour of the transformers. Description of the circuit parameters:

- Nonzero resistance of the windings ($R_1$ and $R_2$) power dissipation through heating, and impedance transformation,
- Frequency and loading dependence of the material permeability $\mu_r=f(freq,B)$,
- Magnetic losses ($R_c$), due to material opposition (friction loss) to orient domain walls according to the external magnetizing field and due to material opposition (conductive loss) to conduct current, which is caused by the induced voltage in the alternating magnetic field.

- Intra-winding capacitance (turn to turn within a winding—$C_{12}$), Inter-winding capacitance (primary to secondary—$C_{1/2}$) Parasitic capacitances limit the upper bandwidth of operation and also reduce the isolation the transformer can provide.

- Finite primary winding inductance ($L_m$) is the magnetising inductance seen from the primary side. For large signal operation – low frequency or/and large amplitude, the core will saturate, and the inductance will change during the course of a voltage cycle. This causes non-linear behaviour, and can lead to transformer failure. It is important to realize that the saturation problem is a function of the applied voltage and frequency only.

- Finite flux capability of the core material, leading to saturation (non-linear behaviour of $I_m$)

- Leakage inductance of the windings ($L_{1/2}$ and $L_{2/1}$)

- The equivalent circuit consists of an ideal transformer of ration $1:n$ that ideally establish the mutual flux.

In the ideal transformer the magnetization is lossless and the number of turns can be easily connected to the selected core cross section $A_c$, peak value of the flux density of the core $B_{cm}$ and the supply voltage $U_1=\omega E$ at frequency $f$ where $\omega=2\pi f$,

$$E_1 = \omega N_1 \Phi = \frac{1}{\sqrt{2}} \omega N_1 B_{cm} A_c$$  \hspace{1cm} (2.14)$$

in the real transformer the magnetic core has a MMF-drop that drags a magnetizing current, which can be related to the operation point of the core at $H_{cm} B_{cm}$,

$$I_{0m} = \frac{\omega N_1}{X_m} \Phi = \frac{1}{\sqrt{2}} \frac{N_1 B_{cm} A_c}{L_m} = \frac{1}{\sqrt{2}} \frac{N_1 B_{cm} A_c I_{path}}{\mu A_c N_1^2} = \frac{1}{\sqrt{2}} \frac{H_{cm} I_{path}}{N_1}$$  \hspace{1cm} (2.15)$$

In addition the magnetization process causes losses due to magnetic hysteresis in the materials and induced electromotive force that is reason for the eddy currents in a conductive material.

$$I_{0e} = \frac{\omega N_1}{R_c} \Phi = \frac{P_{fe}}{\omega N_1 \Phi}$$  \hspace{1cm} (2.16)$$

The equivalent circuit can be further simplified by allowing small calculation inaccuracies without significantly increasing the estimation error.

- The small voltage drop in $R_1$ and $X_1$ compared to $U_1$, and Small magnetizing current $I_0$ in comparison with load current $I_1$ allows the shunt terminals transfer to primary terminals,

- The secondary quantities $R_2$ and $X_2$ may be replaced on the primary side by using ideal transformer property $R_2'=R_2/n^2$ and $X_2'=X_2/n^2$,

- The new equivalent circuit can be used for small power transformer that simplifies the analysis and parameter identification from the experiments.

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2.6 Phasor diagram

With steady state a.c. conditions the instantaneous values can be represent with phasors in frequency domain. The best way to understand things is to draw a phasor diagram of the currents, voltages and fluxes. The primary current is given by magnetizing current and the referred secondary current for a loaded transformer:

\[ I_1 = I_0 + n I_2 \]  

(2.17)

The secondary voltage can be expressed from the equivalent circuit

\[ U_2 = E_2 - I_2 (R_2 + jX_2) = nU_1 - nI_1 (R_1 + jX_1) - I_2 (R_2 + jX_2) \]  

(2.18)

2.7 Solving the equivalent circuit

Based on mathematic expressions on 2.18 and the circuit presentation in Figure 2.1, the output of the transformer is represented on the primary side and the electric equivalent circuit to be solved is presented in Figure 2.4. Similar to the node potential methods that has been used in the thermal equivalent circuit a solution can be obtained to the electric circuit. The electric equivalent circuit (Figure 2.4) consists of complex impedances between the node points.

![Figure 2.3 Simplified equivalent circuit used in practical iron-cored transformers](image)

![Figure 2.4 (Electric) equivalent circuit of a transformer](image)

The Matlab script of the topology matrix is shown below where the element numeration is:

1. \( Z_{1\sigma} \) for the imperfection of the primary coil that includes coil resistance and leakage,
2. \( Z_{2\sigma} \) the secondary coil that is transferred to the primary side,
3. \( Z_2 \) load impedance, that is actually selected as a variable resistive load from idling to short-circuit
4. \( Z_m \) this represents the magnetisation of the magnetic core

\[
\text{Mec} = \begin{bmatrix}
1 & 1 & 2 & \text{R1 + j*2*pi*freq*L1}\_\text{leak} \\
2 & 2 & 3 & \text{R2 + j*2*pi*freq*L2}\_\text{leak} * (\text{N1/N2})^2 \\
3 & 3 & 4 & \text{Rload} * (\text{N1/N2})^2 \\
4 & 4 & 4 & \text{Rm} * j*2*pi*freq*ln / (\text{Rm + j*2*pi*freq}^2) \\
\end{bmatrix};
\]
In order to evaluate the electric potential in the node points the reciprocal of the impedances, admittances, have to be introduced. In the case of simplicity the resistances and the inductances has been used in the topology matrix. The inverse of the load in this code example is the pure conductance without any reactive part. The load can be different from the pure resistance.

The assignment of magnetising characteristics and equivalent transformer circuit is formulated in \texttt{EMK_task_2.m}.

### 2.8 FE model

Finite element model and modelling process is defined in \texttt{EMK_task_2.lua}.

The xy-plane cross-section of a shell type of transformer is the base geometry for the heat transfer analysis. Update the initial data so that it has exactly the same geometry as in the first home assignment.

The modelling process does the following:

1. magnetostatic calculation (frequency=0 Hz) where the estimated magnetisation current is used. The goal is to study the flux and flux density
2. quasistatic electromagnetic calculation (frequency=50 Hz) where the estimated magnetisation current is used. The goal is to study the flux and flux density and estimate transformer parameters. When magnetostatic solver follows the magnetisation curve then the AC solver uses equivalent permeability over the period that is connected to the fundamental value and not to the peak value. This leads to higher permeability, flux and flux density at AC solver than DC solver. For the sake of mathematic formulation and implementation the magnetic permeability should be constant for harmonic problems and therefore there are no differences for linear problems when studying the relation between current and flux.
3. quasistatic electromagnetic calculation at 2 different loading points (frequency=50 Hz) The goal is to study the estimation of currents, fluxes, voltages and estimate the power output according to FEMM (observe results from LUA console)

### 2.9 Assignment

Run \texttt{EMK_task_2}, lua-file by FEMM, m-file by Matlab and follow the information in lua console and Matlab workspace. Copy all the outcome of the provided data into your report (at least 5 figures), study the results and summarise your understanding in the text where you analyse the following.

1. Copy Matlab Figure 1 into your report; express the relation between magnetic field intensity and magnetic flux density in the core. Are you able to motivate the selection of operation point at 1.4 T? What would be the advantages of choosing a lower or a higher magnetic flux density in the core?
2. The size for the transformer and the transforming ratio is selected by you. How do you specify the coils i.e. number of turns and wire diameter? Describe the electromagnetic calculation process of a transformer if you specify the rated output voltage of a transformer. Short description in bullet list is suggested.
3. Focus on the magnetisation of the core: what do you expect to see and what do you actually get from FE models? Study the outcome of the primary and secondary circuit in FEMM. Copy Matlab Figure 2 into your report; select an operation point from analytic
graph and compare it with FE simulations at 0 and 50 Hz. Please consider the quantities

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<thead>
<tr>
<th></th>
<th>Analytic expression</th>
<th>DC FEMM (0Hz)</th>
<th>AC FEMM (50Hz)</th>
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<td>Magnetizing current</td>
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<td>Primary flux linkage</td>
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<td>Secondary flux linkage</td>
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<td>Primary voltage</td>
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<td>Secondary voltage</td>
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<td>Primary inductance</td>
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whether they are average, root mean square (rms) or peak values! Complete table below.

4. Study the power characteristics provided by Matlab (Figure 3) and analyse how well they compare to your expectation based on the thermal calculations and the results the FEMM provides.

5. How the transformer parameters can be calculated from FEMM? Copy Matlab Figure 4 into your report and compare the analytical estimations with the values from FEMM. Try to find out the coupling coefficient $k$ and model a short circuited transformer in FEMM. Include the output image into the report together with current, flux linkage and current amplitudes and amplitudes that you are able to find from fem model and equivalent circuit model.